生命保険数学 問題6　(平成20年11月5日)　(制限時間: 60分)

1. 次の[　　]に当てはまる適切な式、記号又は数値を書け。

(1) ${}_{t|}q_{xy} = {}_tp_{xy} - [\ {}_{t+1}p_{xy}\ ]$

(2) ${}_tq_{\overline{xy}} = ([\ 1\ ] - {}_tp_x)([\ 1\ ] - {}_tp_y)$

(3) ${}_{t|}q_{\overline{xy}} = {}_{t|}q_x + {}_{t|}q_y - [\ {}_{t|}q_{xy}\ ]$

(4) ${}_tp^{[1]}_{\overline{xy}} = {}_tp_x + {}_tp_y - [\ 2\,{}_tp_{xy}\ ]$

(5) $\dfrac{d\,{}_tp_{xy}}{dt} = -[\ {}_tp_{xy}\,\mu_{x+t,y+t}\ ]$

(6) ${}_tp_{\overline{xy}} \cdot \mu_{\overline{x+t,y+t}} = {}_tq_y\,{}_tp_x\,\mu_{x+t} + [\ {}_tq_x\,{}_tp_y\,\mu_{y+t}\ ]$

(7) ${}_{t|}q_{\overset{1}{x}yz} = \int_t^{t+1} {}_sp_{xyz}[\ \mu_{x+s}\ ]\,ds$

(8) ${}_{t|}q_{\overset{2}{x}y} = \int_t^{t+1} [\ {}_sq_y\ ]\,{}_sp_x\,\mu_{x+s}\,ds$

(9) ${}_tq_{\overset{2}{x}y} = \int_0^t {}_sp_{xy}\,\mu_{y+s}[\ {}_{t-s}q_{x+s}\ ]\,ds$

(10) ${}_tq_{\overset{1}{x}y} = {}_tq_x - [\ {}_tq_{\overset{2}{x}y}\ ]$

(11) ${}_tq_{\overset{1}{x}y} - {}_tq_{x\overset{2}{y}} = {}_tp_y[\ {}_tq_x\ ]$

(12) ${}_{t|}q_{\overset{2}{x}y} = {}_{t|}q_{x\overset{1}{y}} + {}_tp_x\,{}_tq_y - [\ {}_{t+1}p_x \cdot {}_{t+1}q_y\ ]$

(13) ${}_{t|}q_{\underset{1}{x}\overset{2}{y}z} = {}_{t|}q_{\overset{1}{y}z} - [\ {}_{t|}q_{x\overset{1}{y}z}\ ]$

(14) $[\ {}_tq_{\underset{1}{x}\underset{2}{y}\overset{3}{z}}\ ] = {}_tq_{\underset{1}{x}\overset{2}{y}z} - {}_tq_{x\overset{2}{y}}\,{}_tp_z$

(15) ${}_{t|}q_{\underset{1}{x}\overset{2:3}{y}z} = {}_{t|}q_{\underset{1}{x}\overset{2}{y}z} + [\ {}_{t|}q_{\underset{1}{x}\underset{2}{y}\overset{3}{z}}\ ]$

(16) ${}_tp_{\overline{xyz}}^{\,2} = {}_tp_{xy} + {}_tp_{yz} + {}_tp_{xz} - [\ 2\,{}_tp_{xyz}\ ]$

(17) ${}_tq_{\overset{3}{x}y\underset{2}{z}\underset{1}{w}} = \int_0^t [\ {}_sq_{\overset{2}{z}w}\ ]\,{}_sp_{xy}\,\mu_{x+s}\,ds$

(18) ${}_{t|}q_{[\ \overset{1}{\overline{xy}},z\ ]} = \int_t^{t+1} {}_sp_{xy}\,{}_sp_z\,\mu_{x+s,y+s}\,ds$

2. 死力 μ_x が $\mu_x = \dfrac{1}{100-x}$ $(0 \le x < 100)$ で与えられるとき、次の値を求めよ。

(19) ${}_{20}q_{\overset{1}{20},40} = \dfrac{5}{24}$

(20) ${}_{20}q_{20,\overset{2}{40},\underset{1}{60}} = \dfrac{5}{72}$

(21) $\mathring{e}_{20,40} = 22.5$

(22) $\mathring{e}_{\overline{20,40}} = 47.5$

3. 死亡法則がゴムパーツの法則 $\mu_x = Bc^x$ に従うとする。次の[　　]に当てはまる適切な c^x, c^y, c^z の式を記入せよ。

(23) ${}_tq_{\overset{1}{x}yz} = \left[\ \dfrac{c^x}{c^x+c^y+c^z}\ \right]{}_tq_{xyz}$

(24) ${}_tq_{\underset{1}{x}\overset{2}{y}z} = \left[\ \dfrac{c^y}{c^y+c^z}\ \right]{}_tq_{yz} - \left[\ \dfrac{c^y}{c^x+c^y+c^z}\ \right]{}_tq_{xyz}$

(25) ${}_\infty q_{\overset{2}{x}yz} = \left[\ \dfrac{c^x}{c^x+c^z} + \dfrac{c^x}{c^x+c^y} - \dfrac{2c^x}{c^x+c^y+c^z}\ \right]$

1 (11)−(14), 2, 3 は pp.2-3 を参照のこと

1. (11) $${}_tq^1_{xy} - {}_tq^2_{xy} = \int_0^t ({}_sp_{xy}\mu_{x+s} - {}_sp_{xy}\mu_{x+s}\,{}_{t-s}q_{y+s})\,ds$$

$$= \int_0^t {}_sp_x\mu_{x+s}\underbrace{{}_sp_y(1-{}_{t-s}q_{y+s})}_{= {}_sp_y\,{}_{t-s}p_{y+s} = {}_tp_y}ds = {}_tp_y\int_0^t {}_sp_x\mu_{x+s}\,ds = {}_tp_y\,{}_tq_x$$

(12) $${}_{t|}q^2_{xy} = {}_{t+1}q^2_{xy} - {}_tq^2_{xy}$$

$$= {}_{t+1}q^1_{xy} - {}_{t+1}p_x\,{}_{t+1}q_y - ({}_tq^1_{xy} - {}_tp_x\,{}_tq_y)\ \cancel{=(\quad)}$$

(11)で x,y を入れ換えたもの

$$= {}_{t|}q^1_{xy} + {}_tp_x\,{}_tq_y - {}_{t+1}p_x\,{}_{t+1}q_y$$

(13) $${}_{t|}q_{x\overset{2}{\underset{1}{y}}z} = \int_t^{t+1} \underbrace{{}_sq_x}_{=1-{}_sp_x}\,{}_sp_y\mu_{y+s}\,{}_sp_z\,ds = {}_{t|}q_{\overset{1}{y}z} - {}_{t|}q_{x\overset{1}{y}z}.$$

(14) $${}_tq_{x\overset{2}{\underset{1}{y}}z} - {}_tq^2_{xy}\,{}_tp_z = \int_0^t {}_sq_x\,{}_sp_y\mu_{y+s}(\underbrace{{}_sp_z - \overset{{}_sp_z\,{}_{t-s}p_{z+s}}{{}_tp_z}}_{= {}_sp_z\,{}_{t-s}q_{z+s}})\,dt$$

$$= \int_0^t {}_sq_x\,{}_sp_{yz}\mu_{y+s}\,{}_{t-s}q_{z+s}\,ds = {}_tq_{x\underset{12}{y}\overset{3}{z}}$$

2. ${}_tp_x = \dfrac{100-x-t}{100-x}$, ${}_tq_x = \dfrac{t}{100-x}$ に注意する。

(19) $${}_{20}q^1_{20,40} = \int_0^{20} {}_sp_{20}\mu_{20+s}\,{}_sp_{40}\,ds = \int_0^{20}\frac{\cancel{80-s}}{80}\,\frac{1}{\cancel{80-s}}\,\frac{60-s}{60}\,ds$$

$$= \frac{1}{80\cdot 60}\Big(\underbrace{60\cdot 20 - \frac12 20^2}_{20\cdot 50}\Big) = \frac{5}{24}$$

(20) $${}_{20}q^2_{20,\underset{1}{40},60} = \int_0^{20} {}_sp_{20}\,{}_sp_{40}\mu_{40+s}\,{}_sq_{60}\,ds$$

$$= \int_0^{20}\frac{80-s}{80}\,\frac{\cancel{60-s}}{60}\,\frac{1}{\cancel{60-s}}\,\frac{s}{40}\,ds$$

$$= \frac{1}{80\cdot 60\cdot 40}\Big(\frac12 80\cdot 20^2 - \frac13 20^3\Big) = \frac{1}{24}\Big(2-\frac13\Big) = \frac{5}{72}$$

(21) $$\mathring{e}_{20,40} = \int_0^\infty {}_tp_{20,40}\,dt = \int_0^{60}\frac{80-t}{80}\,\frac{60-t}{60}\,dt$$

$20s = t$
$$= \int_0^3 \frac{4-s}{4}\,\frac{3-s}{3}\,20\,ds = \frac53\int_0^3(12-7s+s^2)\,ds$$

$$= \frac53\Big(12\cdot 3 - \frac72\cdot 9 + \frac13 3^3\Big) = \frac{45}{2}$$

(22) ${}_tp_{\overline{20,40}} = {}_tp_{20} + {}_tp_{40} - {}_tp_{20,40}$ より

$\mathring{e}_{\overline{20,40}} = \mathring{e}_{20} + \mathring{e}_{40} - \mathring{e}_{20,40} = 40 + 30 - 22.5 = 47.5$ //

注.1 $\mu_x = \dfrac{1}{100-x}$ より x 才の人の余命 X の p.d.f. は

$$f_X(t) = \frac{1}{100-x} \quad (0 \leqq t \leqq 100-x)$$

よって $\mathring{e}_x = E[X] = \dfrac{100-x}{2}$ //

注.2. ${}_tp_{\overline{20,40}} = \begin{cases} 1 - \dfrac{t}{80} \cdot \dfrac{t}{60} & (0 \leqq t \leqq 60) \\ 1 - \dfrac{t}{80} & (60 \leqq t \leqq 80) \end{cases}$ (← 忘れずに!!)

${}_tq_{\overline{xy}} = {}_tq_x\,{}_tq_y$ より

を用いて $\mathring{e}_{\overline{20,40}} = \displaystyle\int_0^{60} \left(1 - \frac{t}{80} \cdot \frac{t}{60}\right) dt + \int_{60}^{80} \left(1 - \frac{t}{80}\right) dt$ と計算しても良い。

3. ~~$\mu_x = Bc^x$ より $\mu_{xyz} = \mu_x + \mu$~~

$\mu_{x+t,y+t,z+t} = \mu_{x+t} + \mu_{y+t} + \mu_{z+t} = Bc^t(c^x + c^y + c^z)$

$= \dfrac{c^x + c^y + c^z}{c^x} \mu_{x+t}$ に注意すると

(23) ${}_tq^1_{xyz} = \displaystyle\int_0^t {}_sp_{xyz}\, \mu_{x+s}\, ds$

$= \displaystyle\int_0^t {}_sp_{xyz} \frac{c^x}{c^x + c^y + c^z} \mu_{x+s,y+s,z+s}\, ds = \frac{c^x}{c^x + c^y + c^z} {}_tq_{xyz}$ //

(24) ${}_tq^{\,2}_{\underset{1}{x}yz} = \displaystyle\int_0^t \underbrace{{}_sq_x}_{=1-{}_sp_x} {}_sp_{yz}\, \mu_{y+s}\, ds = \int_0^t {}_sp_{yz}\, \mu_{y+s}\, ds - \int_0^t {}_sp_{xyz}\, \mu_{y+s}\, ds$

$= \displaystyle\int_0^t {}_sp_{yz} \frac{c^y}{c^y + c^z} \mu_{y+s,z+s}\, ds - \int_0^t {}_sp_{xyz} \frac{c^y}{c^x + c^y + c^z} \mu_{x+s,y+s,z+s}\, ds$

$= \dfrac{c^y}{c^y + c^z} {}_tq_{yz} - \dfrac{c^y}{c^x + c^y + c^z} {}_tq_{xyz}$

(25) ${}_\infty q^{\,2}_{xyz} = {}_\infty q^{\,2}_{x\underset{1}{y}z} + {}_\infty q^{\,2}_{xy\underset{1}{z}}$

$\overset{(24)}{=} \dfrac{c^x}{c^x + c^z} \underbrace{{}_\infty q_{xz}}_{=1} - \dfrac{c^x}{c^x + c^y + c^z} \underbrace{{}_\infty q_{xyz}}_{=1} + \dfrac{c^x}{c^x + c^y} \underbrace{{}_\infty q_{xy}}_{=1} - \dfrac{c^x}{c^x + c^y + c^z} \underbrace{{}_\infty q_{xyz}}_{=1}$

(いつか死ぬ。)

$= \dfrac{c^x}{c^x + c^z} + \dfrac{c^x}{c^x + c^y} - \dfrac{2c^x}{c^x + c^y + c^z}$ //