生命保険数学 問題3　(平成20年 ~~9~~ 10 月 ~~27~~ 15 日)　(制限時間: 30分)

1. 次の空欄に当てはまる適切な式、記号又は数値を書け。

(1) $[\ p_{x+t}\ ] = \exp\left(-\int_t^{t+1} \mu_{x+s}\, ds\right)$

(2) $a_{x:\overline{n}|} = \sum_{t=1}^{n} [\ a_{\overline{t-1}|}\ ] \cdot {}_{t-1|}q_x + [\ a_{\overline{n}|}\ ] \cdot {}_np_x$

(3) ${}_mP^{1}_{x:\overline{n}|} = \dfrac{A^{1}_{x:\overline{n}|}}{[\ \ddot{a}_{x:\overline{m}|}\ ]}$

(4) $A^{1}_{x:\overline{n}|} = 1 - d\,\ddot{a}_{x:\overline{n}|} - [\ v^n{}_np_x\ ]$

(5) $[\ M_x\ ] = vN_x - N_{x+1}$

(6) $A^{1}_{x:\overline{n}|} = v \cdot [\ \ddot{a}_{x:\overline{n}|}\ ] - a_{x:\overline{n}|}$

(7) $[\ P_{x:\overline{n}|}\ ] = \dfrac{1}{\ddot{a}_{x:\overline{n}|}} - d$

(8) $1 = \dfrac{1}{A_{x:\overline{n}|}} - \dfrac{d}{[\ P_{x:\overline{n}|}\ ]}$

(9) $\overline{A}_{x:\overline{n}|} = 1 - [\ \delta\ ] \cdot \overline{a}_{x:\overline{n}|}$

(10) $[\ {}_m\overline{P}^{(\infty)}_{x:\overline{n}|}\ ] = \dfrac{\overline{A}_{x:\overline{n}|}}{\overline{a}_{x:\overline{m}|}}$ (記号を書け)

(11) $\dfrac{1-(1+i)A_x}{1-A_{x+1}} = [\ p_x\ ]$ (*p.2)

(12) $\dfrac{A_{x+n}-A_x}{1-A_x} + \dfrac{\ddot{a}_{x+n}}{\ddot{a}_x} = [\ 1\ ]$ (*p.2)

(13) $\sum_{t=1}^{\infty} l_{x+t}A_{x+t} = l_x \cdot [\ a_x\ ]$

(14) ${}_tV_{x:\overline{n}|} = [\ A_{x+t:\overline{n-t}|}\ ] - P_{x:\overline{n}|} \cdot [\ \ddot{a}_{x+t:\overline{n-t}|}\ ]$ (将来法)

(15) ${}_tV_{x:\overline{n}|} = P_{x:\overline{n}|} \cdot \left[\ \dfrac{N_x - N_{x+t}}{D_{x+t}}\ \right] - \left[\ \dfrac{M_x - M_{x+t}}{D_{x+t}}\ \right]$ (過去法, 計算基数で表せ)

(16) ${}_tV_{x:\overline{n}|} = 1 - \dfrac{[\ \ddot{a}_{x+t:\overline{n-t}|}\ ]}{\ddot{a}_{x:\overline{n}|}}$

(17) ${}_tV_{x:\overline{n}|} = \dfrac{P_{x:\overline{n}|} - P^{1}_{x:\overline{t}|}}{[\ P_{x:\overline{t}|}^{\ \ 1}\ ]}$ (p.2)

(18) ${}_{t-1}V_{x:\overline{n}|} + [\ P_{x:\overline{n}|} - v\,q_{x+t-1}\ ] = v\,p_{x+t-1}\,{}_tV_{x:\overline{n}|}$

(19) 養老保険の第 t 年度における貯蓄保険料は $[\ v\,{}_tV_{x:\overline{n}|} - {}_{t-1}V_{x:\overline{n}|}\ ]$.

(20) 養老保険の第 t 年度における危険保険料は $[\ v\,q_{x+t-1}(1 - {}_tV_{x:\overline{n}|})\ ]$.

(21) $m < n$ のとき、${}_tV_{x:\overline{m}|} - {}_tV_{x:\overline{n}|} = (P_{x:\overline{m}|} - P_{x:\overline{n}|}) \cdot \left[\ \dfrac{N_x - N_{x+t}}{D_{x+t}}\ \right]$. (計算基数で表せ)

2. 次を計算基数を用いて表せ。

(22) $\overline{A}_{x:\overline{n}|} = \dfrac{\overline{M}_x - \overline{M}_{x+n} + D_{x+n}}{D_x}$

(23) $\overline{P}_{x:\overline{n}|} = \dfrac{\overline{M}_x - \overline{M}_{x+n} + D_{x+n}}{N_x - N_{x+n}}$

(24) $(D\ddot{a})_{x:\overline{n}|} = \dfrac{nN_x - (S_{x+1} - S_{x+n+1})}{D_x}$ (p.2)

(25) $(I_{\overline{n}|}A)_x = \dfrac{R_x - R_{x+n}}{D_x}$ (p.2)

(11) $(\text{与式}) = \dfrac{1-(\cancel{1+i})(\cancel{v}q_x + \cancel{v}p_x A_{x+1})}{1-A_{x+1}} = \dfrac{p_x - p_x A_{x+1}}{1-A_{x+1}} = p_x$ //

(12) $(\text{与式}) = \dfrac{\cancel{1}-d\ddot{a}_{x+n} - (\cancel{1}-d\ddot{a}_x)}{\cancel{1}-(\cancel{1}-d\ddot{a}_x)} + \dfrac{\ddot{a}_{x+n}}{\ddot{a}_x} = \dfrac{\ddot{a}_x - \ddot{a}_{x+n}}{\ddot{a}_x} + \dfrac{\ddot{a}_{x+n}}{\ddot{a}_x} = 1$ //

(13) $\displaystyle\sum_{t=1}^{\infty} l_{x+t} A_{x+t} = \sum_{t=1}^{\infty}\sum_{s=1}^{\infty} v^s \underbrace{l_{x+t}\cdot {}_{s-1|}q_{x+t}}_{= d_{x+t+s-1}}$

$\displaystyle = \sum_{s=1}^{\infty} v^s \underbrace{\sum_{t=1}^{\infty} d_{x+t+s-1}}_{= d_{x+s} + d_{x+s+1} + \cdots = l_{x+s}} = l_x \sum_{s=1}^{\infty} v^s {}_s p_x = l_x a_x$ //

(17) (15) より ${}_tV_{x:\overline{n}|} = P_{x:\overline{n}|}\dfrac{1}{\dfrac{D_{x+t}}{N_x - N_{x+t}}} - \dfrac{\dfrac{M_x - M_{x+t}}{N_x - N_{x+t}}}{\dfrac{D_{x+t}}{N_x - N_{x+t}}} = P_{x:\overline{n}|}\dfrac{1}{P_{x:\overline{t}|}^{\ \ 1}} - \dfrac{P^1_{x:\overline{t}|}}{P_{x:\overline{t}|}^{\ \ 1}}$ //

(24) $(D\ddot{a})_{x:\overline{n}|} = n + (n-1)v p_x + \cdots + 2v^{n-2}{}_{n-2}p_x + v^{n-1}{}_{n-1}p_x$

$= \dfrac{1}{D_x}(nD_x + (n-1)D_{x+1} + \cdots + 2D_{x+n-2} + D_{x+n-1})$

$= \dfrac{1}{D_x}(D_x + D_{x+1} + \cdots + D_{x+n-2} + D_{x+n-1}) \rightsquigarrow N_x - N_{x+n}$

$+ D_x + D_{x+1} + \cdots + D_{x+n-2} \rightarrow N_x - N_{x+n-1}$

$+ D_x + D_{x+1} \rightsquigarrow N_x - N_{x+2}$

$+ D_x) \rightsquigarrow N_x - N_{x+1}$

$= \dfrac{1}{D_x}(nN_x - (N_{x+1} + \cdots + N_{x+n})) = \dfrac{1}{D_x}(nN_x - (S_{x+1} - S_{x+n+1}))$ //

(28) $(I_{\overline{n}|}A)_x = v q_x + 2\,{}_{1|}q_x + \cdots + (n-1)\,{}_{n-2|}q_x + n\,{}_{n-1|}q_x + n\,{}_{n|}q_x + \cdots$

$= \dfrac{1}{D_x}(C_x + 2C_{x+1} + \cdots + (n-1)C_{x+n-2} + nC_{x+n-1} + nC_{x+n} + \cdots)$

$= \dfrac{1}{D_x}(M_x + M_{x+1} + \cdots + M_{x+n-2} + M_{x+n-1}) = \dfrac{R_x - R_{x+n}}{D_x}$ //